

## A LINEAR PROGRAMMING FUEL MANAGEMENT MODEL FOR THE HTGR

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**Abstract**—Linear programming was used as the optimization technique to minimize the amount of U-235 used by an example 1160 MWe HTGR during three and six burn periods while simultaneously maximizing the production of U-233. The reactor core was divided into four concentric annular zones; an "out-in" fuel movement technique was used while the fissile loading of the core was held uniform by adjusting the power peaking constraints. The reactor was linearized by holding the neutron flux constant over each of the burn periods. The model was used to consider three cases: Case 1 consisted of three burn periods with no U-233 recycle, Case 2 consisted of six burn periods with no U-233 recycle, and Case 3 consisted of six burn periods with U-233 recycle allowed at the fourth refuelling event. The results indicate that the amount of U-233 produced in the first eight years of operation of the 1160 MWe HTGR will be sufficient to operate the same reactor with no new U-235 fuel for 3 yr hence.

### INTRODUCTION

The possibility of gaining simultaneously energy and new fuel is of great importance for the economics of power stations, especially in light of current energy shortages. Many feel that the High-Temperature Gas-Cooled Reactor (HTGR) concept will be the major thermal reactor of the future (Stewart, 1971; Coiby, 1971; USAEC, 1969; Miller, 1974). Electric utilities are now beginning to accept the HTGR design; hence, fuel management techniques for the HTGR should be more widely studied.

The major goal of fuel management is the complete administration and control of technical and economic factors to optimize the use of nuclear fuel to achieve the highest performance at a minimum cost (Glasstone, 1967). Indeed, the value of the fuel used in an HTGR over its lifetime is usually about twice the capital costs of the plant (Asmussen, 1972). Many methods have been developed to optimize the combinations of fuel inputs to meet operational needs (Clark, 1971; Tabak, 1968; Brown, 1973; Chen, 1973; Suzuki, 1971; Savar, 1971). Of these optimization methods "linear programming" offers much promise. All previously reported studies have used either a boiling water reactor (BWR) or a pressurized water reactor (PWR) as the reference reactor. To date no reported studies have been made of the HTGR core using linear programming (LP) as a means of optimization. Therefore, linear programming was used in this work as the optimization technique to find the

optimum combination of U-233 fuel produced and U-235 fuel used to breed U-233.

The main purpose of this work was to develop a versatile LP model which has incorporated appropriate but simple burnup equations and operational constraints to simulate a HTGR core. This simple model could serve as a preliminary projector of fuel needs and fuel loading patterns for a nuclear utility company operating a HTGR.

### THEORY

In a classical sense, optimization has been defined as the art of obtaining best policies to satisfy certain objectives and simultaneously to satisfy fixed requirements (Gothfried, 1973). Therefore, optimization of a process requires the development of a mathematical model, i.e. the appropriate equations written to establish mathematical cause-and-effect relationships. In this work the model consists of simple transmutation equations (burnup equations) and all requirements needed to keep the HTGR operational; the best policy in this case was to get the most productivity for the least expense, i.e. an optimum combination between U-235, U-233, and Th-232.

In many instances the objectives of an optimization problem cannot be stated easily, since it involves the minimization or maximization of some function of dissimilar entities (Gothfried, 1973; Taka, 1971). U-235 and U-233, two dissimilar entities, are related. U-235 may be purchased at a specified cost, while U-233 is produced from Th-232 by a nuclear conversion process; thus, the value of

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U-233 is its neutronic worth relative to that of U-235, and expressed by the U-233 indifference value (Asmussen, 1972). The analysis of a reactor core requires that the constraints be formulated in such a manner that the desired power level is obtained while thermal-hydraulic, power-peaking, and criticality requirements are not violated.

#### Linear programming theory.

Detailed LP (Linear Programming) theory and analysis may be found in many references (Gothfried, 1973, Taka, 1971, Gass, 1969, Hadley, 1963); therefore, briefly stated the LP problem is to find a vector  $\{X\}$  which minimizes or maximizes the objective function,  $Z$ ,

$$Z = [C]\{X\} \quad (1)$$

subject to the constraints

$$\{X\} \geq \{O\} \quad (2)$$

and

$$(A)\{X\} (\geq, =, \text{ or } \leq) \{B\} \quad (3)$$

where

$[C]$  is a  $1 \times n$  matrix of constants,

$\{X\}$  is an  $n \times 1$  matrix of decision variables,

$\{O\}$  is a null column vector,

$(A)$  is an  $m \times n$  coefficient matrix,

$\{B\}$  is an  $m \times 1$  requirements matrix.

In equation (3) only one of the ( $\leq$ ,  $=$ , or  $\geq$ ) can apply for each equation.

#### MODEL FORMULATION

The HTGR fuel consists of a mixture of fissionable U-235, U-233, Pu-239 and Pu-241 and fertile (Th-232 and U-238) isotopes. Since the principal products of a power reactor are thermal energy and isotope production resulting from the irradiation of the fuel, the main consideration in simulating a nuclear reactor must be given to the formulation of the equations which predict the formation and depletion of the major nuclides in the core of the reactor, the burnup equations. The thermal energy produced by the operating reactor is obtained directly from the burnup equations.

#### Burnup equations

The composition of the fuel will change during irradiation, because of the depletion of initial fissionable material, buildup of fission products, and the buildup of nuclides in fissionable and fertile material. The change in composition of the

fuel is proportional to the concentration per unit volume of the nuclides present and to the neutron flux. There are many nuclide chain reactions taking place in a nuclear reactor, and to simulate them all would be an almost insurmountable task; therefore, reactor designers often only consider the major chains which contain the most significant fissile and fertile materials.

In this work the only nuclides considered were Th-232, U-233, U-235, U-236, U-238, Pu-239, Pu-240, Pu-241, Pu-242, fission products, Xe-135 and Sm-149.

Through the capture of a neutron, U-238 first transmutes into U-239 which is in turn transformed into Pu-239 by double beta-decay. Some of the nuclides produced with a mass number 239 are lost because Np-239, with an absorption cross-section of about 60 barns (Shulten, 1967), can also absorb neutrons, being transformed into Np-240. Since the latter and its successors are not significantly fissile the transmutation into Np-240 represents a small but genuine loss of neutrons. The conversion of Np-239 into Np-240 was neglected in this work. In general, the neutrons lost through the capture of a neutron in Pa-233 is not large enough to make a significant change in the neutron cycle. Although for some power histories Pa-233 can be important, this isotope was neglected in this work.

The general form of the burnup equation is:

$$\dot{N} = AN, \quad (4)$$

where

$\dot{N}$  is the derivative with respect to time of the time dependent nuclide densities (state variables),  $N$ ,

$A$  is an  $n \times n$  matrix of constants which depend principally on the neutron cross sections of the isotopes considered in the model,

$n$  is the number of isotopes (or state variables) considered in the model.

The nuclide densities satisfy the initial conditions

$$N|_{t=0} = N^0 \quad (5)$$

The solution of equation (4) found after applying the initial conditions of equation (5) is:

$$N = BN^0 \quad (6)$$

where  $B$  is the time dependent state transition matrix.

An analytical form for equation (6) is possible when both the neutron flux and the microscopic cross sections are constant during the fuel irradiation (Blomstrand, 1968). In this work the neutron flux level and the microscopic cross sections were held constant throughout the life of each burn period.

summation signs,  $R$ ,  $A$ ,  $D$  and  $C$  refer to the number of core regions, the number of fissile isotopes, the total of fertile plus fissile species, and the number of other absorber species. The superscript 0 refers to BOL and  $N_{gq}^0 \sigma_g^a$  is the macroscopic absorption cross section for carbon in the  $g$ -th core region at BOL.

To obtain the maximum performance, the EOL reactivity was set equal to zero; therefore, the EOL reactivity constraint is

$$0 = \left( \sum_q^R \sum_g^A \nu_g N_{gq}^1 \sigma_g^f \right) P_{a11} - \left\{ \sum_j^D \left[ \sum_p N_{pq}^1 \sigma_p^a + N_{cq}^1 \sigma_c^a + \sum_j N_{jq}^1 \sigma_j^a \right] + \sum_g^A N_{gq}^1 \sigma_g^f Y_g^P + \frac{\phi_q \bar{\sigma}_a^X}{\lambda^X + \bar{\sigma}_a^X \phi_q} \times \sum_g^A N_{gq}^1 \sigma_g^f (Y_g^I + Y_g^X) + N_{B10}^1 \sigma_{B10}^a \right\} \quad (11)$$

where all terms are the same as for equation (10) but the superscript 1 refers to EOL.

Since the total reactivity constraints only require that the overall system be critical, "localized reactivity" constraints are also required to assure a reasonable loading pattern. To have flexibility in the LP model while using the "localized reactivity" constraints and to prevent severe flux tilting as a result of an azimuthal reactivity imbalance (FSAR, 1973; Van Howe, 1964) the infinite multiplication factor was used to constrain the BOL of each region.

Thus the "localized reactivity" constraint for each region is given by

$$0 < \left( \sum_g^A \nu_g N_g^0 \sigma_g^f \right) p\epsilon - \left[ \sum_p^D N_p^0 \sigma_p^a + N_c^0 \sigma_c^a + \sum_j^C N_j^0 \sigma_j^a \right] + \sum_g^A N_g^0 \sigma_g^f Y_g^P + \frac{\phi_q \bar{\sigma}_a^X}{\lambda^X + \bar{\sigma}_a^X \phi_q} \times \sum_g^A N_g^0 \sigma_g^f (Y_g^I + Y_g^X) + N_{B10}^0 \sigma_{B10}^a \quad (12)$$

#### Total power constraints

The power in a reactor is proportional to the product of the flux and the macroscopic fission cross

sections. The BOL total power constraint can be given in a linear equation as

$$\text{TPOW} \times \text{PFAC} < \left( \sum_q^R V_q \phi_q \sum_g^A \text{REC}_g N_{gq}^0 \sigma_g^f \right) F_{PC} \quad (13)$$

where PFAC is the load factor and TPOW is the total power requirements of the HTGR in MWt,  $\phi_q$  is the average neutron flux in the  $q$ th region;  $V_q$  is the active core volume in the  $q$ th region;  $\text{REC}_g$  is the recoverable energy per fission of the  $g$ th isotope (MeV/fission) in the  $q$ th region;  $\sigma_g^f$  is the microscopic fission cross section for the  $g$ th species;  $F_{PC}$  is the factor used to convert MeV per sec to megawatts (MW).

Similarly, the EOL total power constraint can be given as

$$\text{TPOW} \times \text{PFAC} < \left( \sum_q^R V_q \phi_q \sum_g^A \text{REC}_g N_{gq}^1 \sigma_g^f \right) F_{PC} \quad (14)$$

#### Power peaking constraints

The "power peaking" constraints serve two purposes in this analysis: (1) to control the distribution of the macroscopic fission cross section which, if not constrained, leads to excessive peak-to-average power ratios (Brown, 1973) and (2) to insure a fuel loading pattern consistent with the flux shape assumption.

"Power peaking" constraints were established in each region in this model and at the BOL and EOL. The BOL "power peaking" constraint can be given as an inequality by

$$P_r P_{wtq} P_{avg} > \left( \phi_q \sum_g^A \text{REC}_g N_{gq}^0 \sigma_g^f \right) F_{PP} \quad (15)$$

where  $P_r$  is the radial power peaking factor,  $P_{wtq}$  is the weighting factor for the  $q$ th region  $P_{avg}$  is the average power density (W/cc), and  $F_{PP}$  is the factor used to convert (MeV/sec) to MW.

The EOL "power peaking" constraint can be given in a similar manner as

$$P_r P_{wtq} P_{avg} > \left( \phi_q \sum_g^A \text{REC}_g N_{gq}^1 \sigma_g^f \right) F_{PP} \quad (16)$$

#### Volume constraints

To calculate the maximum space permissible for the fuel in the HTGR, both sizes of fuel particles TRISO, which contains the uranium and the BISO, which contains the thorium must be considered.

The LP inequality for each region scaled by  $10^{-27}$  is given by

$$1 > \frac{1}{4630} N_{02}^0 + \frac{1}{1160} N_{25}^0 + \frac{1}{1160} N_{28}^0. \quad (17)$$

where  $4630 \times 10^{27}$  and  $1160 \times 10^{27}$  are the maximum number of Th and U atoms per region, respectively. For the burnable poison, the LP constraint scaled by  $10^{-27}$  is given by

$$1 > \frac{1}{550} N_{B10}^0 \quad (18)$$

where  $N_{B10}^0$  is the initial load of B-10 in each region and  $550 \times 10^{27}$  is the maximum number of B-10 atoms per region.

#### Enrichment constraint

The optimum enrichment for the 1160 MWe power reactor is 93% (Colby, 1971; USAEC, 1969). However, in this analysis the enrichment was allowed to vary. The LP enrichment constraint is given by

$$N_{25q}^0(1 - En_{\max}) - N_{23q}^0 En_{\max} < 0 \quad (19)$$

where  $En_{\max}$  is 0.93.

#### Objective function

The objective function was formulated on the basis that the U-233 indifference value can be used to equate the cost relationship between U-233 and U-235. From results based on 20 year levelized fuel cycle cost calculations, the U-233 indifference value was found to be about \$16.7/g U-233 (Asmussen, 1972). Since the price of highly enriched uranium is approximately \$12/g U-235, the comparative cost ratio between U-233 and U-235 is 1.39. Hence, the objective function for the HTGR is given by

$$\text{Minimize: } OB = \sum_n^R (N_{23q}^0 - 1.39 N_{25q}^0) \quad (20)$$

where  $N_{25q}^0$  is the initial load concentration in the  $q$ th region, and  $N_{23q}^1$  is the EOL concentration of U-233 in the  $q$ th region. This objective function serves two purposes: It minimizes the amount of U-235 loaded into the system, and it simultaneously maximizes the amount of U-233 produced by the system.

#### DATA FOR THE MODEL

Table 1 is a listing of typical characteristics of an example HTGR (patterned after the Fort St. Vrain facility). Because of the proprietary nature of detailed characteristics the commercial HTGR,

Table 1. HTGR Characteristics.

Power level, MW thermal	2900
MW electrical	1160
Efficiency, %	40
Inlet gas temperature, °F	640
Outlet gas temperature, °F	1430
Core configuration	
Active height, ft.	20.8
Equivalent Diameter, ft.	27.6
Active volume, cc $\times 10^7$	35.71
Reflector thickness, ft.	3 to 4
Number of control rods	73 pairs
Fuel Management	
Fuel lifetime at 80% capacity factor	4
Fraction of core replaced each cycle	1/4
Number of refueling regions	73
Fuel elements	
Number of elements	3944
Number of columns	493
Element height, in.	31.2
Element width, in.	14.2
Fuel exposure	(across flats)
Average MWD/MT	90000
Peak fast fluence, $10^{21}$ nvt	8

some data used in the model had to be assumed. Tables 2-4 contain data which were used in the model. Table 5 contains various fuel management parameters adopted for use in this work.

#### SCENARIOS

Two fuel cycles, a reference thorium cycle with recycle (U-235-U-233/Th-232/U-233) and a thorium cycle without recycle (U-235/Th-232/U-233), were considered in this work. The following modes of operation result from consideration of these two fuel cycles.

(1) *Nonrecycle Operation.* Fuel charged to the reactor consists of highly enriched uranium and thorium. The spent fuel removed from the core is either sold or placed in storage awaiting reprocessing and recycle.

(2) *Initial recycle operation.* An interim period for the early HTGR's when the stored U-233 is used exclusively to fuel the reactor.

(3) *Recycle operation.* The fuel removed from the core is reprocessed and the U-233 is recycled into the reactor along with highly enriched uranium makeup.

Table 2. HTGR core physics parameters.

Microscopic cross sections, barns ( $T_n = 1624^{\circ}\text{F}$ )	
$\sigma_a$ (U-233)	183
$\sigma_a$ (Th-232)	1.59
$\sigma_a$ (U-238)	9.33
$\sigma_a$ (U-235)	156
$\sigma_a$ (U-236)	10.9
$\sigma_a$ (U-234)	26.3
$\sigma_a$ (Pu-239)	614
$\sigma_a$ (Pu-240)	508
$\sigma_a$ (Pu-241)	451
$\sigma_a$ (FP of U-235)	19.3/fission
$\sigma_a$ (FP of U-233)	16.3/fission
$\sigma_a$ (SM-149)	$1.06 \times 10^4$
$\sigma_a$ (Xe-135)	$3.74 \times 10^3$
$\sigma_a$ (graphite)	$5.90 \times 10$
$\sigma_a$ (B-10)	$5.49 \times 10^2$
Capture to fission ratio	
$\alpha_{25}$	.224
$\alpha_{23}$	.150
$\alpha_{49}$	.620
$\alpha_{41}$	.440
Average yield of neutrons per fission	
$\nu_{25}$	2.43
$\nu_{23}$	2.50
$\nu_{49}$	2.89
$\nu_{41}$	3.03
Miscellaneous	
Fast fission factor	1.015
Resonance escape probability (Th-232)	.70
Resonance escape probability (U-238)	.986
Fission-to-resonance nonleakage	.98
Total nonleakage	.97
$\phi_{\text{max}}$	$2.33 \times 10^{14}$
	n/cm <sup>2</sup> sec
REC 23	193 MeV/fission
REC 25	204 MeV/fission
REC 49	213 MeV/fission
$P_r$ (radial power peaking factor)	1.6

## RESULTS AND DISCUSSION

Three cases were considered in this work:

Case 1—Three burn periods with no recycle capabilities, (U-235/Th-232/U-233),

Case 2—Six burn periods with no recycle capabilities,

Case 3—Six burn periods with U-233 recycle capabilities in the fourth refuelling event i.e. for the first three years the cycle can be given as (U-235/Th-232/U-233), and for the next three years the cycle can be given as (U-235-U-233/Th-232/U-233).

For these three cases the optimum mass balance was obtained using IBM Mathematical Programming System (IBM-MPS). The results are presented in Tables 6, 7 and 8. With the exception of Th-232 concentration, the initial input concentrations of the first burn period of both Case 1 and Case 2 is approximately the same. The larger input concentration of Th-232 in Case 1 can be attributed to the less restrictive overall reactivity requirements of Case 1. The fuel loaded into the system for Case 1

has to meet only three overall reactivity requirements whereas the fuel loaded into the system for Case 2 has to meet four overall reactivity requirements; therefore, a heavier loading of Th-232 exists in the initial core of Case 1. Also less fissile material was needed in Case 1, Cycle 3 than was needed in Case 2, Cycle 3. The smaller amount of fissile material loading is due to the larger fertile (Th-232) loading of the fuel in the initial two cycles of Case 1. Since a larger fertile loading was present in Case 1, more U-233 was bred in the first 3 cycles of Case 1 than was bred in the first 3 cycles of Case 2; therefore, there was less U-235 needed to meet the operational requirements of the third cycle of Case 1.

Table 3. Fission product yields (atoms/fission) from thermal fission (taken from Lamarsh, 1966).

Fissionable Isotope	<sup>233</sup> U	<sup>235</sup> U	<sup>239</sup> Pu
Yield of <sup>135</sup> I (direct)	.051	.061	.055
Yield of <sup>135</sup> Xe (direct)		.003	
Yield of <sup>149</sup> Pm (indirect)	.0066	.0113	.019

<sup>a</sup>Taken from (Lamarsh, 1966)

Table 4. Constants for fission product poisoning calculations (decay constants, Lamarsh, 1966, and cross sections, Pfeiffer, 1973).

Isotope	$\lambda$ (sec <sup>-1</sup> )	$\bar{\sigma}_a$ (barns)
<sup>135</sup> I	$2.87 \times 10^{-5}$	
<sup>135</sup> Xe	$2.09 \times 10^{-5}$	$374. \times 10^3$
<sup>149</sup> Pm	$3.56 \times 10^{-6}$	
<sup>149</sup> Sm	(stable)	$106. \times 10^2$

Table 5. Fuel management parameters.

Fuel Cycle	Thorium/Uranium
Fuel lifetime (nominal)	4 years at 80% capacity factor
Replaceable reflector lifetime (nominal)	8 years
Average power density	8.4 W/cc
Refueling interval	1 year at 80% capacity factor
Average C/Th ratio	250
Flux ratio (average to maximum) for each region	0.88, 0.66, 0.48, 0.31

Table 6. Mass balance, thorium 3 cycle without recycle.

Fueling event	1	2	3	
Time, years	0	1	2	3
<u>Fresh makeup, kg-atoms</u>				
Th-232	35500.	36500.	10700.	
U-235	1780.	520.	152.	
U-238	132.	38.7	11.3	
<u>Recycle feed, kg-atoms</u>				
U-233		0.	0.	0.
<u>Discharge, kg-atoms</u>				
Th-232		8270.	7080.	6520.
U-233		125.	177.	204.
U-234		4.64	12.0	18.6
U-235		270.	193.	155.
U-236		28.2	41.9	49.9
U-238		24.3	19.5	16.5
Pu-239		2.76	3.20	3.32
Pu-240		.726	1.18	1.37
Pu-241		.326	.587	.706
<u>Optimum Power Levels</u>				
MWt	2320	2406	2370	

<sup>a</sup>At .8 load factor

The total power constraint had to be lowered in the third and fourth cycles of operation to obtain a feasible solution for Case 2 and 3. This lowering of the power requirements was necessary because of the interaction between the shuffling schemes used the coupling effects of the system. The "out-in" shuffling technique is very restrictive since the fuel loaded into core region 4 (outside region) must eventually traverse the complete core, region-by-region. This means the fuel loaded into the initial

Table 7. Mass balance, thorium 6 cycle without recycle.

Fueling event	1	2	3	4	5	6	
Time, years	0	1	2	3	4	5	6
<u>Fresh makeup, (kg-atoms)</u>							
Th-232	24300.	3870.	410.	10800.	36500.	13100.	
U-235	1800.	466.	490.	455.	520.	187.	
U-238	132.	34.7	36.4	36.1	35.7	13.9	
<u>Recycle feed (kg-atoms)</u>							
U-233	0.	0.	0.	0.	0.	0.	
<u>Discharge (kg-atoms)</u>							
Th-232		8720.	10600.	3070.	768.	3480.	641.
U-233		126.	192.	187.	191.	204.	187.
U-234		4.67	12.9	17.4	21.3	22.3	21.0
U-235		268.	188.	161.	146.	141.	148.
U-236		26.3	40.8	52.1	59.4	57.0	60.0
U-238		24.2	18.6	17.8	17.1	16.0	17.5
Pu-239		2.76	3.26	3.26	3.28	3.34	3.27
Pu-240		.724	1.19	1.37	1.46	1.45	1.45
Pu-241		.324	.577	.724	.794	.776	.800
<u>Optimum Power Levels</u>							
MWt	2320	2326	2260	2240	2149	2370	

<sup>a</sup>At .5 load factor

core has a restrictive effect on any fuel loaded into the core within the next four years. In Cycle 3 of Case 2 the total power produced is only 2260 MWt. This total power produced is only about 2.5% lower than the power required in the other cycles. This low power level may be caused by constraints on either side of Cycle 3, i.e. to satisfy the "power peaking" and reactivity constraints in Cycle 2 and Cycle 5.

It was found that the constraint of major importance was the "power peaking" constraint in region 1 (centre region). This constraint actually dictated the fissile and fertile fuel loaded in the other regions since an out-in fuel shuffling technique was employed.

Predictions indicate that after eight years the accumulated U-233 from previous burn periods

Table 8. Mass balance, thorium 6 cycle with U-238 recycle.

Fueling event	1	2	3	4	5	6	
Time, years	0	1	2	3	4	5	6
<u>Fresh makeup (kg-atoms)</u>							
Th-232	24300.	4320.	1010.	237.	1780.	34300.	
U-235	1780.	464.	486.	0.	0.	0.	
U-238	132.	34.5	36.1	1510.	1836.	2590.	
<u>Recycle feed (kg-atoms)</u>							
U-233	0.	0.	0.	409.	499.	.706.	
<u>Discharge (kg-atoms)</u>							
Th-232		8640.	10600.	3520.	376.	3620.	641.
U-233		125.	192.	189.	199.	206.	190.
U-234		4.67	12.9	21.1	21.1	22.5	21.2
U-235		269.	188.	160.	147.	140.	147.
U-236		28.1	40.8	56.2	59.7	56.8	59.5
U-238		24.2	18.6	17.6	17.3	15.8	17.2
Pu-239		2.76	3.26	3.27	3.28	3.33	3.28
Pu-240		.724	1.19	1.37	1.46	1.47	1.48
Pu-241		.323	.577	.722	.800	.772	.800
<u>Optimum Power Levels</u>							
MWt	2320	2320	2260	2260	2408	2810	

<sup>a</sup>At .8 load factor

would furnish all of the fissile requirements of the HTGR for the following three years, resulting in good neutronics and a high conversion ratio (Dahlberg, 1970). This situation was simulated but on a smaller scale with Case 3. In Case 3, the U-233 was held out of the system for 3 yr; then it was allowed as recycle feed for use in the 4th refuelling event. Four hundred and nine (409) kg-atoms were needed to refuel the reactor, and no U-235 was taken into the system at the 4th refuelling event. The 409 kg-at actually surpassed the amount of U-233 produced in the first two years of operation, i.e. 317 and 318 kg-at produced in the first two years of operation in

Case 3 and in Case 2, respectively. Further, if the amount of U-233 produced for eight years is extrapolated from Case 2, the total amount of U-233 was 1461 kg-at. If 450 kg-at are taken as the average amount of U-233 to refuel the reactor each year, i.e. 1350 kg-at for 3 yr, the results of this work verify the prediction of eight years operation followed by 3 yr of independent operation on U-233 recycle. Actually, after eight years of operation the next 3 yr of operation are not entirely independent of U-235 since U-235 will be in the system in varying amounts for at least four more years.

In summary, the model formulated in this work is a reasonable beginning for future HTGR fuel management techniques. The two main constraints that control the model are the "power peaking" constraint in region 1 and the EOL overall reactivity constraint. These constraints, however, could lose their dominant influence on the problem by changing the shuffling technique, and relaxing the EOL overall reactivity constraint.

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